Prolog
Programming in Logic

Lecture #6

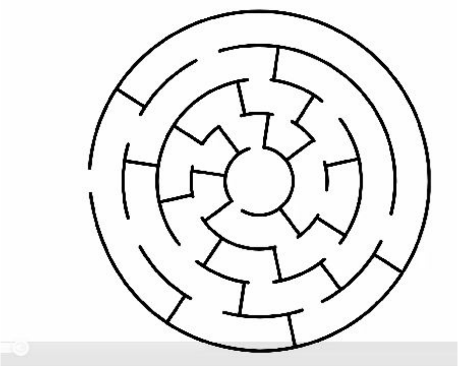
Ian Lewis, Andrew Rice

Today's discussion

Videos

Countdown

Graph search



Q: You mentioned that we can use cuts and negation in the exam. Can we also use implication (->)?

Q: You mentioned that we can use cuts and negation in the exam. Can we also use implication (->)?

A: No. You also can't use ';', assume any library predicates, or use any extra-logical stuff (except cut) like findAll, call etc.

Q: When figuring out what a Prolog program does, how can we work out which of the arguments are intended to be supplied with constants, and which with variables.

A: Did I manage to answer this last time?

```
% foo(+X,-Y) succeeds if output number Y
% is double input number X
foo(X,Y) :- Y is 2 * X.
```

Q: What does Prolog allow us to do (other than coding in a different way) that other languages can't? Not meaning to sound dismissive just curious of applications!

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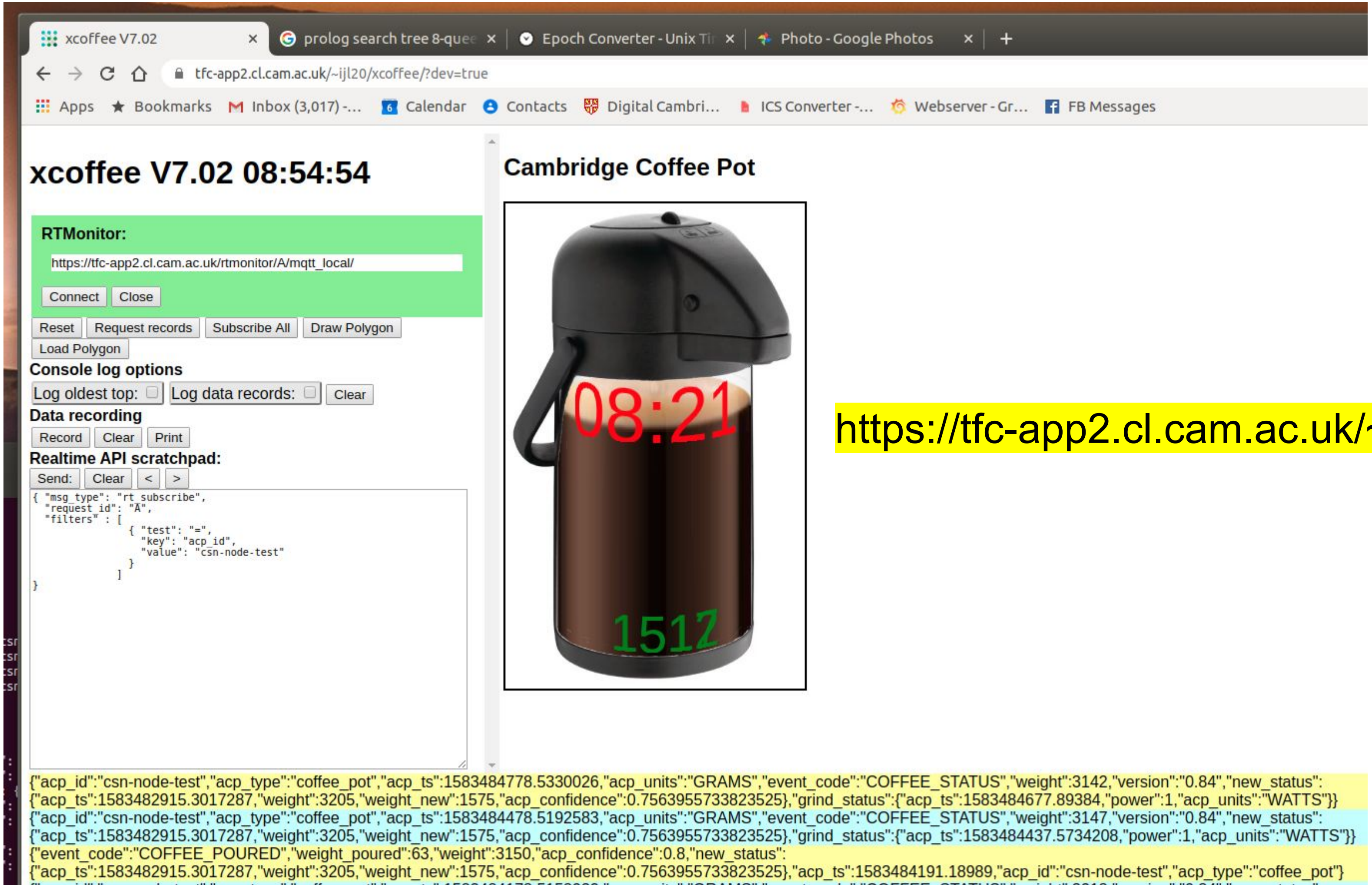
A:

* Pure Prolog subset 'Datalog' used for network verification.
* Prolog used for Java Virtual Machine verification
* Prolog quite good at digital logic simulation
... theorem provers written in Prolog.

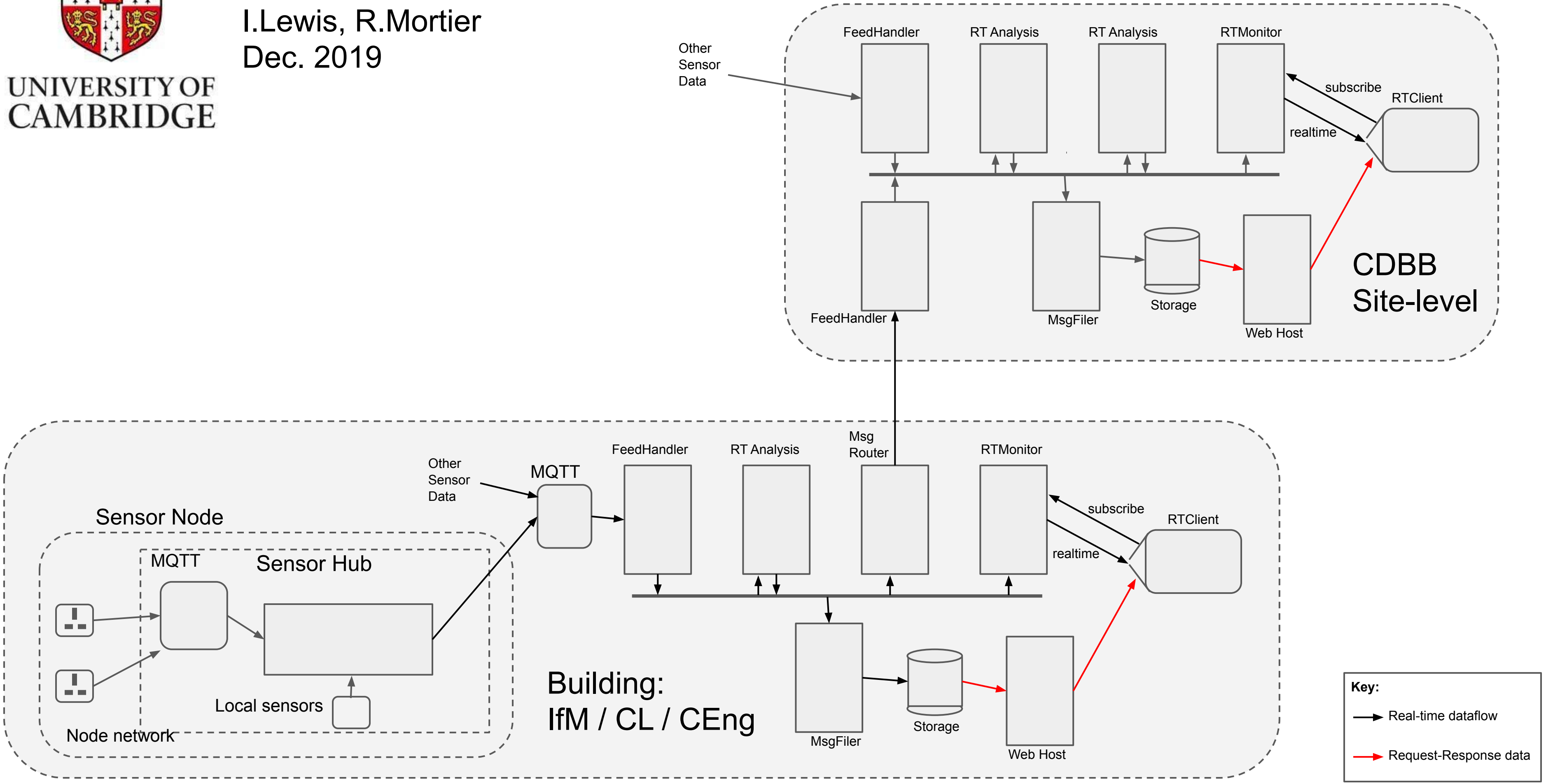
* Sooner or later, some method of reasoning with NN data will emerge.



https://tfc-app2.cl.cam.ac.uk/~ijl20/xcoffee

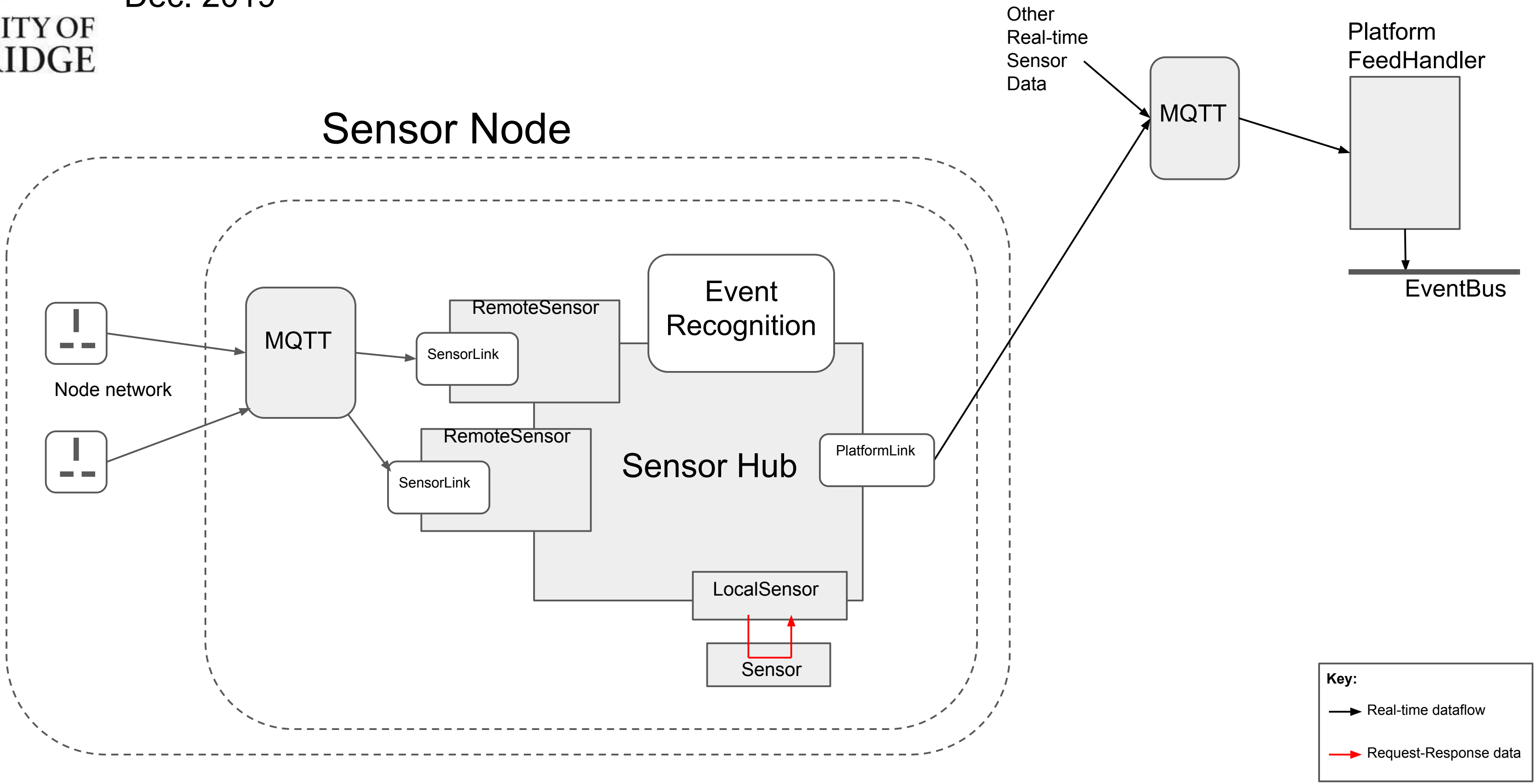
CDBB Digital Architecture for Real-Time Data

I.Lewis, R.Mortier
Dec. 2019

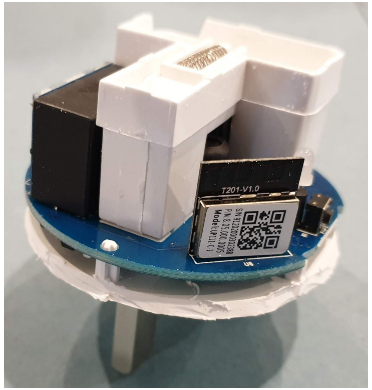


CDBB Sensor Node Architecture

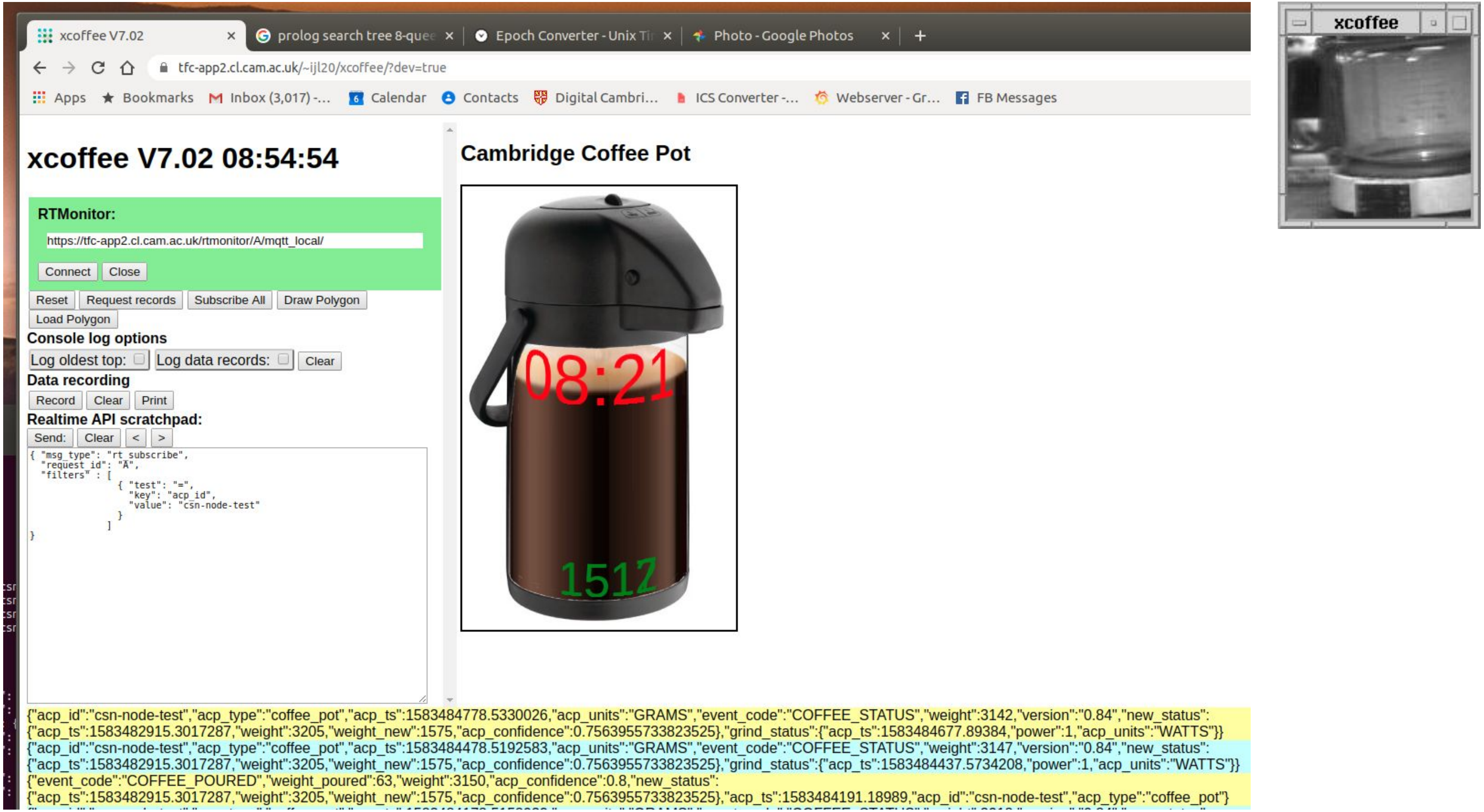
I.Lewis, R.Mortier
Dec. 2019



Sensors producing *real-time data*




bodum
PASS
SANTOS
How to make a pot of coffee:


xcoffee V7.02
prolog search tree 8-quee
Epoch Converter - Unix Ti
Photo - Google Photos
tfc-app2.cl.cam.ac.uk/~ijl20/xcoffee/?dev=true
Apps
Bookmarks
Inbox (3,017) -...
Calendar
Contacts
Digital Cambri...
ICS Converter -...
Webserver - Gr...
FB Messages
xcoffee V7.02 08:54:54
RTMonitor:
https://tfc-app2.cl.cam.ac.uk/rtmonitor/A/mqtt_local/
Connect
Close
Reset
Request records
Subscribe All
Draw Polygon
Load Polygon
Console log options
Log oldest top:
Log data records:
Clear
Data recording
Record
Clear
Print
Realtime API scratchpad:
Send:
Clear
<
>
{ "msg_type": "rt_subscribe",
"request_id": "A",
"filters" : [
{ "test": "=",
"key": "acp_id",
"value": "csn-node-test"
}
]
}
Cambridge Coffee Pot
08:21
1512
xcoffee
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{"acp_ts":1583482915.3017287,"weight":3205,"weight_new":1575,"acp_confidence":0.7563955733823525},"grind_status":{"acp_ts":1583484677.89384,"power":1,"acp_units":"WATTS"}}
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CDBB Digital Architecture for Real-Time Data

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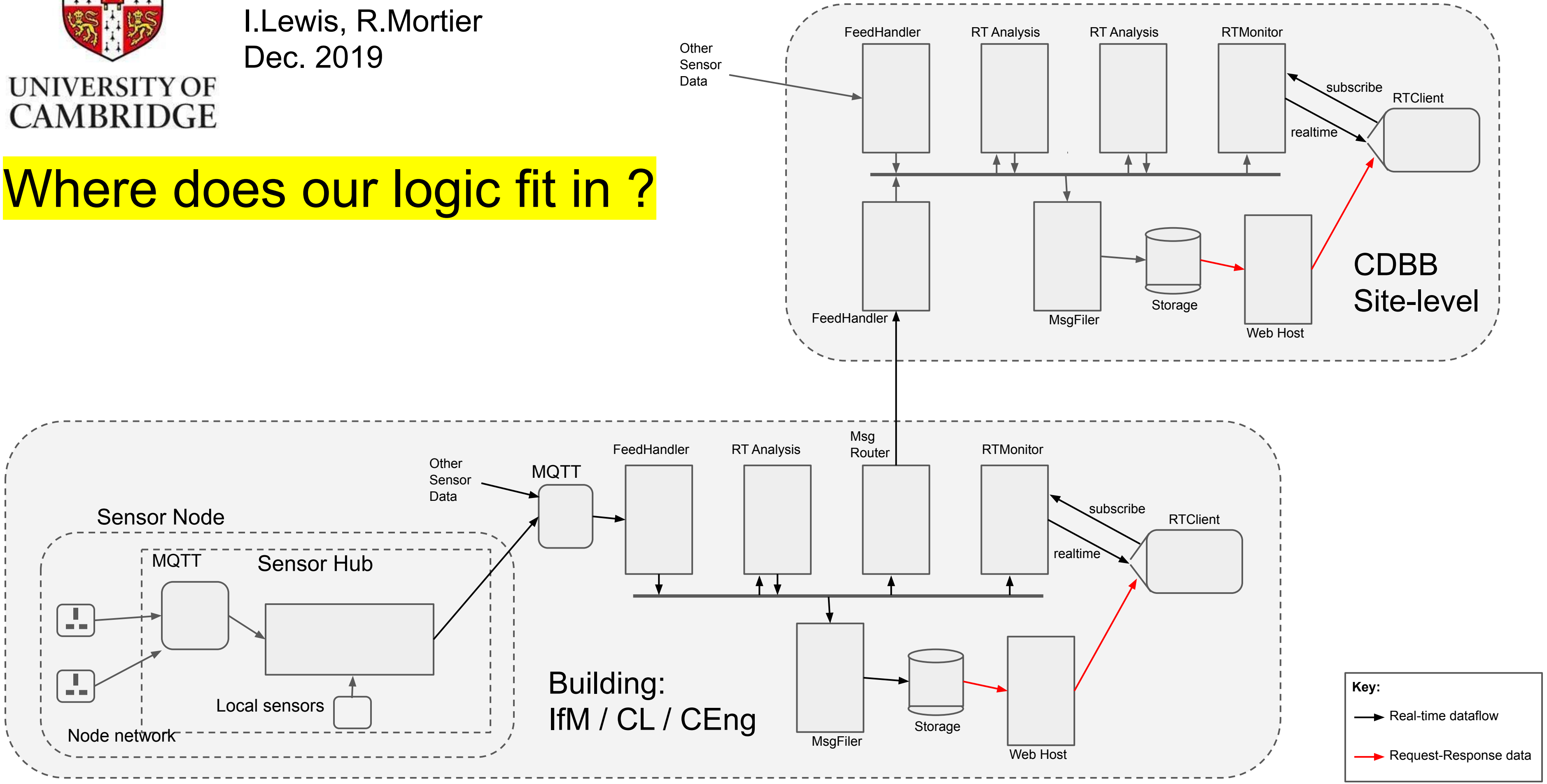
Where does our logic fit in ?



Q: Operators & precedence? :- op(700, xfx, arc).

300	xfx	mod	Arithmetic function
400	yfx	*	Arithmetic function
400	yfx	/	Arithmetic function
400	yfx	//	Arithmetic function
500	fx	+	Arithmetic function
500	fx	-	Arithmetic function
500	yfx	+	Arithmetic function
500	yfx	-	Arithmetic function
700	xfx	<	Predicate
700	xfx	=	Predicate
700	xfx	=..	Predicate
700	xfx	<	Predicate
700	xfx	>	Predicate
700	xfx	>=	Predicate
700	xfx	is	Predicate
900	fy	\+	Predicate
1000	xfy	,	Predicate
1100	xfy	;	Predicate
1200	fx	:-	Introduces a directive
1200	xfx	:-	head :- body. separator

1. Precedence 0..1200 (0 highest)
2. (...) has precedence 0
3. :- and , both have low precedence so you can have

   (complicated stuff) :- (more complicated stuff), ... , ...
4. . is an "end delimiter"

fx	Prefix (non-associative).
fy	Prefix (right-associative) e.g. fact fact 3.
xfx	Infix (non-associative)
xfy	Infix (right-associative)
yfx	Infix (left-associative)

Q: Operators & precedence?

```
arc(X,Y)
op(700, xfx, arc).
```

Used 700 because that's typical for a relation (aka Predicate)
Used xfx because we won't have `A arc B arc C`.

```
a arc b.
b arc c.
c arc d.
c arc e.

path(A,B) :- A arc B.
path(A,B) :- A arc X, path(X,B).
```

Countdown

Countdown [25, 50, 75, 100, 3, 6], Target 952

Start with 6 values: [25, 50, 75, 100, 3, 6]

Remove any 2 values (e.g. [75, 3]) and generate symbolic formula for this pair, add to head of remaining list, e.g.

[(75+3), 25, 50, 100, 6] (note list now of length 5)

If head of list evaluates to 952: SUCCESS

else repeat, e.g. new pair [(75 + 3), 100], (leaves [25, 50, 6])

generate new operator for pair (e.g. +): [(75 + 3) + 100, 25, 50, 6] (length 4)

Countdown [25, 50, 75, 100, 3, 6], Target 952

```
countdown([Soln|_],Target, Soln) :-  eval(Soln,Target).

countdown(L,Target,Soln) :-  choose(2,L,[A,B],R), arithop(A,B,C), countdown([C|R],Target,Soln).
```

generate pair,
generate arithmetic op on pair
    Solution?
    generate pair
    generate arithmetic op on pair
        Solution?
        generate pair
        generate arithmetic op on pair
            Solution?
            ...

choose

```
% choose(N, List, Chosen, Remaining)
choose(0,L,[],L).
choose(N,[H|T],[H|C], Remaining) :- N > 0, M is N-1, choose(M,T,C,Remaining).
choose(N,[H|T],Chosen,      [H|S]) :- N > 0, choose(N,T,Chosen,S).
```

choose

```
% choose(N, List, Chosen, Remaining)
choose(0,L,[],L).
choose(N,[H|T],[H|C], Remaining) :- N > 0, M is N-1, choose(M,T,C,Remaining).
choose(N,[H|T],Chosen,      [H|S]) :- N > 0, choose(N,T,Chosen,S).
```

Base case - choose zero from list L, Chosen = [ ], Remaining = L.

choose

```
% choose(N, List, Chosen, Remaining)
choose(0,L,[],L).
choose(N,[H|T],[H|C], Remaining) :- N > 0, M is N-1, choose(M,T,C,Remaining).
choose(N,[H|T],Chosen,     [H|S]) :- N > 0, choose(N,T,Chosen,S).
```

Base case - choose zero from a list L, Chosen = [ ], Remaining = L.

First recursive case: choose Head, choose N-1 from Tail

choose

```
% choose(N, List, Chosen, Remaining)
choose(0,L,[],L).
choose(N,[H|T],[H|C], Remaining) :- N > 0, M is N-1, choose(M,T,C,Remaining).
choose(N,[H|T],Chosen,     [H|S]) :- N > 0, choose(N,T,Chosen,S).
```

Base case - choose zero from a list L, Chosen = [ ], Remaining = L.

First recursive case: choose Head, choose N-1 from Tail

Seconds recursive case: ignore Head, choose N from Tail, Remaining = H + remaining from tail.

choose

An aside/caution regarding functional support...

```
% choose(N, List, Chosen, Remaining)
choose(0,L,[],L).
choose(N,[H|T],[H|C], Remaining) :- N > 0, choose(N-1,T,C,Remaining).
choose(N,[H|T],Chosen,     [H|S]) :- N > 0, choose(N,T,Chosen,S).
```

E.g. also:

```
... , take(max(L), L, Remaining) , ...
```

choose

An aside/caution regarding functional support...

```
% choose(N, List, Chosen, Remaining)
choose(0,L,[],L).
choose(N,[H|T],[H|C], Remaining) :- N > 0, M is N-1, choose(M,T,C,Remaining).
choose(N,[H|T],Chosen,     [H|S]) :- N > 0, choose(N,T,Chosen,S).
```

F L A T T E N I N G

E.g. also:

```
... , max(L,M), take(M, L, Remaining) , ...
```

Does choose look familiar to you ?

choose

```
% choose(N, List, Chosen, Remaining)
choose(0,L,[],L).
choose(N,[H|T],[H|C], Remaining) :- N > 0, M is N-1, choose(M,T,C,Remaining).
choose(N,[H|T],Chosen,     [H|S]) :- N > 0, choose(N,T,Chosen,S).
```

For our purposes choose/4 could be choose/3...

choose is basically take: -- I've swapped arguments around, keeping you on your toes...

```
take(H,[H|T],T).
take(X,[H|T],[H|R]) :- take(X,T,R).
```

Alternative version of choose

```
% choose(N, List, Chosen, Remaining)
choose(0, L, [], L).
choose(N,[H|T], [H|C], Remaining) :- N > 0, M is N-1, choose(M,T,C,Remaining).
choose(N,[H|T], Chosen, [H|S]) :- N > 0, choose(N,T,Chosen,S).
```

choose is basically take:

```
take(H, [H|T], T).
take(X, [H|T], [H|R]) :- take(X,T,R).
```

E.g. we can write a take_list(A,B,C):

```
% take_list(+A,+B,-C) succeeds if list C is the remaining elements from B after removing list A.
% call with A instantiated to a list of variables, and B ground.
take_list([ ], L, L).
take_list([H|T],L,R) :- take(H,L,LR), take_list(T, LR, R).

?- take_list([A,B], [a,b,c], L).
A=a, B=b, L = [c]
```

eval : reducing arithmetic terms to a number.

```
countdown([Soln|_],Target, Soln) :-  eval(Soln,Target).

countdown(L,Target,Soln) :-  choose(2,L,[A,B],R), arithop(A,B,C), countdown([C|R],Target,Soln).
```

generate pair,
generate arithmetic op on pair
    Solution?
    generate pair
    generate arithmetic op on pair
        Solution?
        generate pair
        generate arithmetic op on pair
            Solution?
            ...

eval : reducing arithmetic terms

```
% eval(+ArithTerm, -N)
eval(A+B,C) :- eval(A,A1), eval(B,B1), C is A1 + B1.
eval(A*B,C) :- eval(A,A1), eval(B,B1), C is A1 * B1.
eval(A/B,C) :- eval(A,A1), eval(B,B1), C is A1 / B1.
eval(A-B,C) :- eval(A,A1), eval(B,B1), C is A1 - B1.
eval(A,A) :- number(A).
```

I'm showing an alternative to Andy's plus(A,B) etc. terms, simply to show infix operators +, -, *, / which already conveniently have the required precedence.

Can you spot anything here?

eval : reducing arithmetic terms

```
% eval(+ArithTerm, -N)
eval(A+B,C) :- eval(A,A1), eval(B,B1), C is A1 + B1.
eval(A*B,C) :- eval(A,A1), eval(B,B1), C is A1 * B1.
eval(A/B,C) :- eval(A,A1), eval(B,B1), C is A1 / B1.
eval(A-B,C) :- eval(A,A1), eval(B,B1), C is A1 - B1.
eval(A,A) :- number(A).
```

I'm showing an alternative to Andy's plus(A,B) etc. terms, simply to show infix operators +, -, *, / which already conveniently have the required precedence.

? Did you spot this alternative implementation:

```
eval(ArithTerm, N) :- N is ArithTerm.
```

arithop - generating arithmetic expressions

```
% arithop(+A, +B, -ArithTerm)
arithop(A,B,A+B).
arithop(A,B,A-B) :- eval(A,D), eval(B,E), D>E.
arithop(B,A,A-B) :- eval(A,D), eval(B,E), D>E.
arithop(A,B,A*B) :- eval(A,D), D \== 1, eval(B,E), E \== 1.
arithop(A,B,A/B) :- eval(B,E), E \== 1, E \== 0, eval(A,D), 0 is D rem E.
arithop(B,A,A/B) :- eval(B,E), E \== 1, E \== 0, eval(A,D), 0 is D rem E
```

We're only generating arithmetic terms relevant the the puzzle, i.e. we're using the result of the eval to check the term.

* There's a minor detail/choice here, whether the 'choose' generates both pairs (e.g. 3,4 and 4,3) or this can be provided by arithop as we are doing here.

Countdown - alternative version of countdown/3

Current version:

```
countdown([Soln|_],Target, Soln) :-  eval(Soln,Target).

countdown(L,Target,Soln) :-  choose(2,L,[A,B],R),
                             arithop(A,B,C),
                             countdown([C|R],Target,Soln).
```

Countdown - alternative version of countdown/3

```
countdown([Soln|_],Target, Soln) :-  eval(Soln,Target).

countdown(L,Target,Soln) :-  choose(2,L,[A,B],R),
                             arithop(A,B,C),
                             countdown([C|R],Target,Soln).


test(Soln,Target,Soln) :- eval(Soln,Target).

countdown(L,Target,Soln) :- take_list([A,B], L, R),
                            arithop(A,B,C),
                            ( test(C, Target, Soln) ; countdown([C|R],Target, Soln) ).
```

Countdown - alternative version of countdown/3

```
countdown([Soln|_],Target, Soln) :-  eval(Soln,Target).

countdown(L,Target,Soln) :-  choose(2,L,[A,B],R),
                             arithop(A,B,C),
                             countdown([C|R],Target,Soln).


test(Soln,Target,Soln) :- eval(Soln,Target).

countdown(L,Target,Soln) :- take_list([A,B], L, R),
                            arithop(A,B,C),
                            test_or_calc(C,Target,Soln,R).

test_or_calc(C,Target,Soln,_) :- test(C, Target, Soln).
test_or_calc(C,Target,Soln,R) :- countdown([C|R],Target, Soln) .
```

Countdown - alternative version of countdown/3

```
countdown([Soln|_],Target, Soln) :-  eval(Soln,Target).

countdown(L,Target,Soln) :-  choose(2,L,[A,B],R),
                             arithop(A,B,C),
                             countdown([C|R],Target,Soln).


test(Soln,Target,Soln) :- eval(Soln,Target).

countdown(L,Target,Soln) :- take_list([A,B], L, R),
                            arithop(A,B,C),
                            ( test(C, Target, Soln) ; countdown([C|R],Target, Soln) ).
```

Countdown Iterative Deepening

The whole point of this section is that you understand *how/why* to apply iterative deepening, rather than assume a specific implementation.

```
test(Soln,Target,Soln) :- eval(Soln,Target).

countdown(L,Target,Soln) :- take_list([A,B], L, R),
                            arithop(A,B,C),
                            ( test(C, Target, Soln) ;
                              countdown([C|R],Target, Soln) ).
```

Countdown Iterative Deepening

```
diff(A,B,Diff) :- Delta is A - B, (Delta < 0 , Diff is -Delta ; Delta >= 0, Diff is Delta).

test(Soln,Target,Soln, Threshold) :- eval(Soln,Result), diff(Result,Target,Diff), Diff =< Threshold.

countdown(L,Target,Soln, Threshold) :- take_list([A,B], L, R),
                                        arithop(A,B,C),
                                       ( test(C, Target, Soln, Threshold) ;
                                        countdown([C|R],Target, Soln, Threshold) ).
```

We add a 'Threshold' to the search clause, implement a 'diff' function, test succeeds within bounds.

Diff =< Threshold: the approach is slightly different here than in the video (both are valid) - we are asking for solutions *within* a 'distance' from the exact answer (not *at* an exact distance).

Countdown Iterative Deepening

```
diff(A,B,Diff) :- Delta is A - B, (Delta < 0 , Diff is -Delta ; Delta >= 0, Diff is Delta).

test(Soln,Target,Soln, Threshold) :- eval(Soln,Result), diff(Result,Target,Diff), Diff =< Threshold.

countdown(L,Target,Soln, Threshold) :- take_list([A,B], L, R),
                                        arithop(A,B,C),
                                       ( test(C, Target, Soln, Threshold) ;
                                        countdown([C|R],Target, Soln, Threshold) ).
```

```
:- countdown([25,50,75,100,3,6],952,Soln,5)
```

Countdown Iterative Deepening

```
diff(A,B,Diff) :- Delta is A - B, (Delta < 0 , Diff is -Delta ; Delta >= 0, Diff is Delta).

test(Soln,Target,Soln, Threshold, Diff) :- eval(Soln,Result), diff(Result,Target,Diff), Diff =< Threshold.

countdown(L,Target,Soln, Threshold, Diff) :- take_list([A,B], L, R),
                                              arithop(A,B,C),
                                             ( test(C, Target, Soln, Threshold, Diff) ;
                                              countdown([C|R],Target, Soln, Threshold, Diff) ).
```

Countdown Iterative Deepening

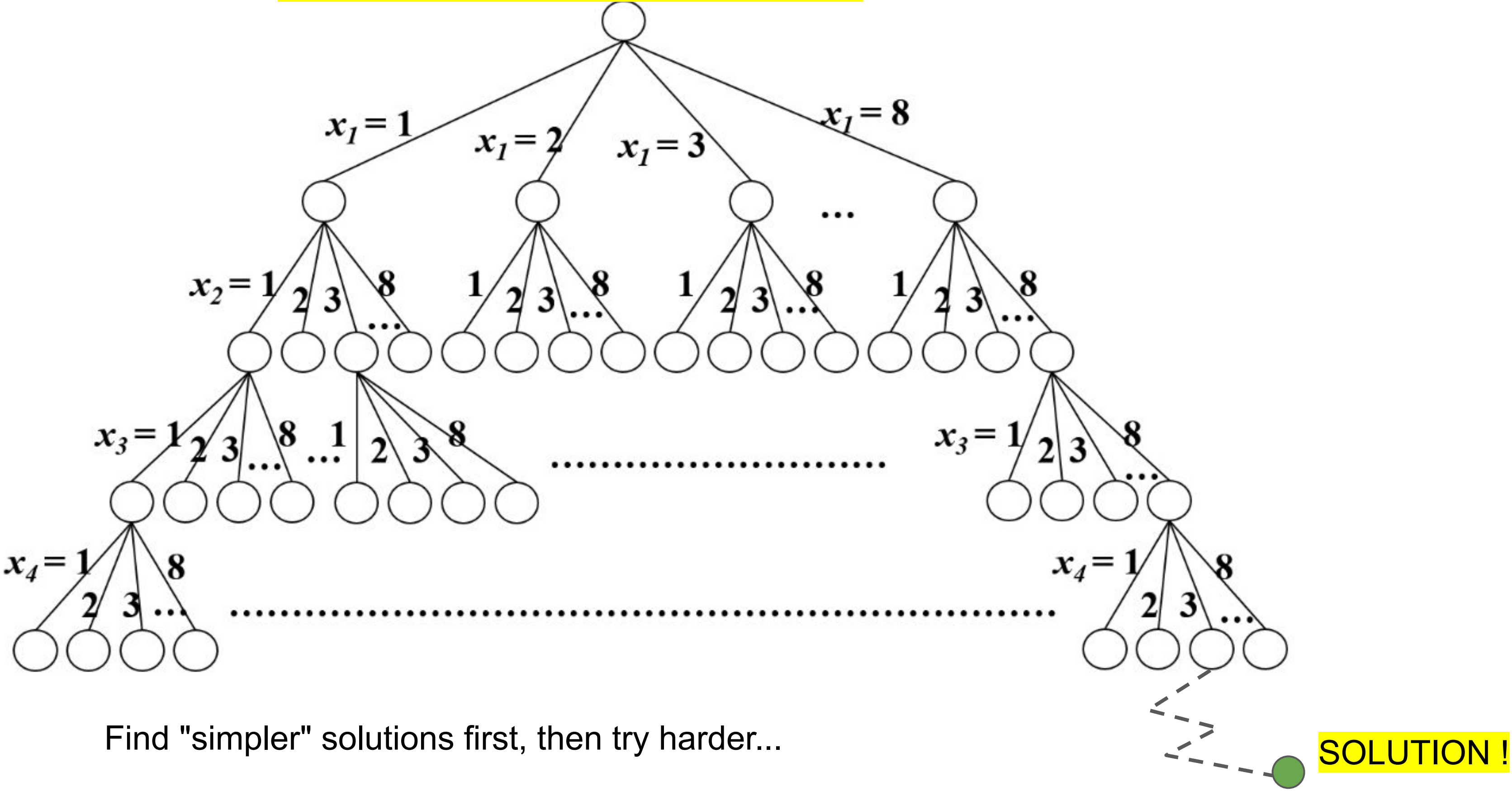
```
diff(A,B,Diff) :- Delta is A - B, (Delta < 0 , Diff is -Delta ; Delta >= 0, Diff is Delta).

test(Soln,Target,Soln, Threshold, Diff) :- eval(Soln,Result), diff(Result,Target,Diff), Diff =< Threshold.

countdown(L,Target,Soln, Threshold, Diff) :- take_list([A,B], L, R),
                                             arithop(A,B,C),
                                            ( test(C, Target, Soln, Threshold, Diff) ;
                                             countdown([C|R],Target, Soln, Threshold, Diff) ).
```

Required Threshold   Actual Difference

```
:- countdown([25,50,75,100,3,6],952,Soln,5, Diff)
```

..EXAMPLE

Countdown Iterative Deepening



Find "simpler" solutions first, then try harder...

Countdown Iterative Deepening - Conclusion

```
diff(A,B,Diff) :- Delta is A - B, (Delta < 0 , Diff is -Delta ; Delta >= 0, Diff is Delta).

test(Soln,Target,Soln, Threshold, Diff) :- eval(Soln,Result), diff(Result,Target,Diff), Diff =< Threshold.

countdown(L,Target,Soln, Threshold, Diff) :- take_list([A,B], L, R),
                                             arithop(A,B,C),
                                            ( test(C, Target, Soln, Threshold, Diff) ;
                                             countdown([C|R],Target, Soln, Threshold, Diff) ).
```

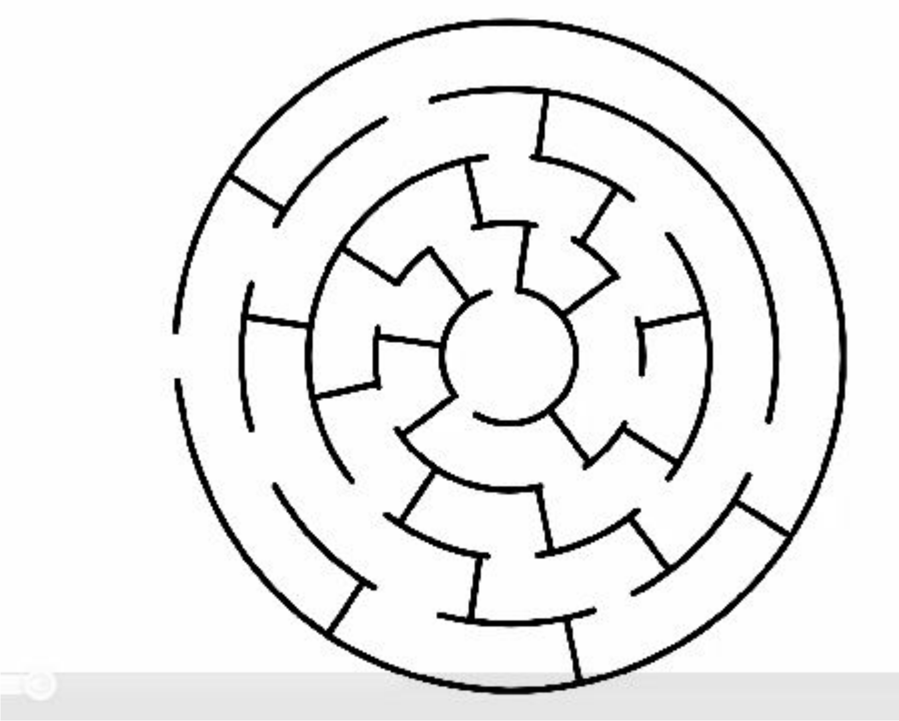
Required Threshold   Actual Difference

```
:- countdown([25,50,75,100,3,6],952,Soln,5, Diff)
```

Summary: use-case can be "find solution within threshold, check difference, find better solution ..."

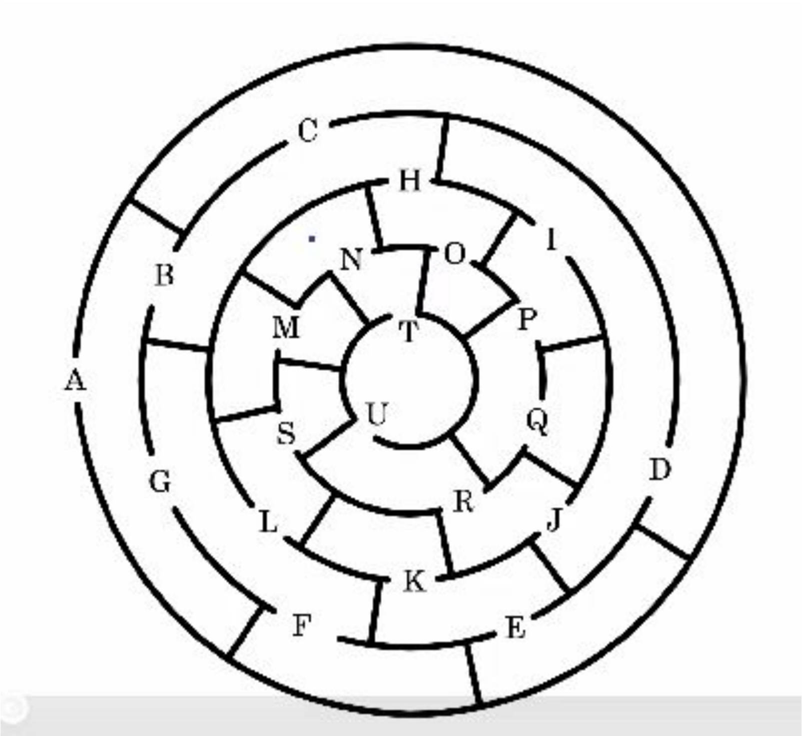
Also as video: `closest(L, Target, Soln, Threshold) :- range(0,100,Threshold), solve2(L,Target,Soln,Threshold).`

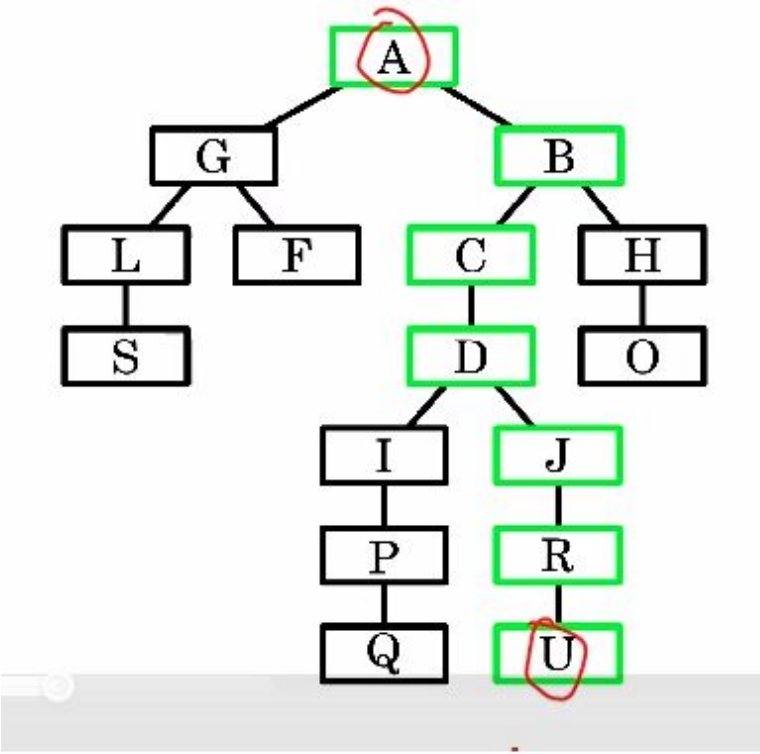
Graph Search



Problem statement

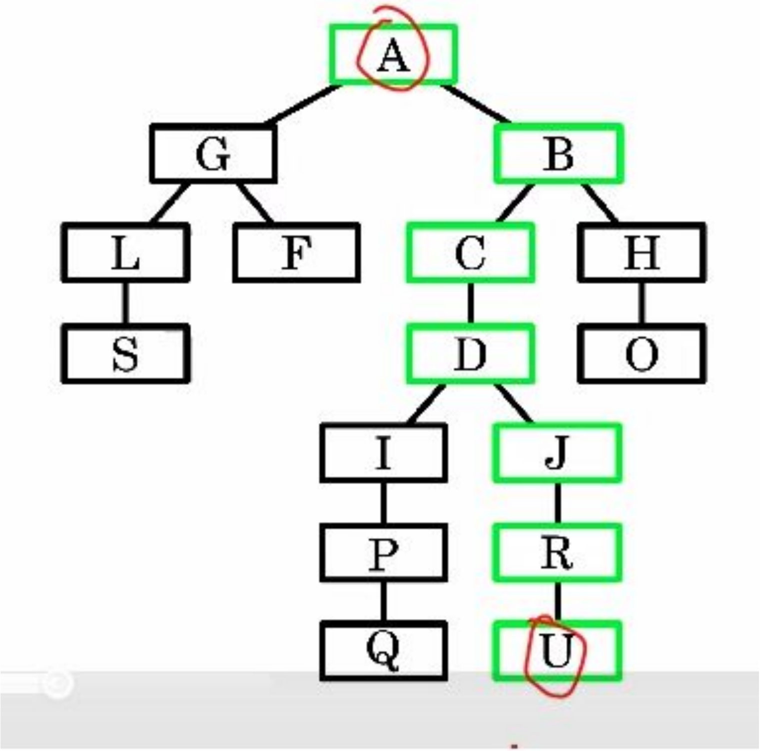
Graph Search





Convert to graph...

Graph Search

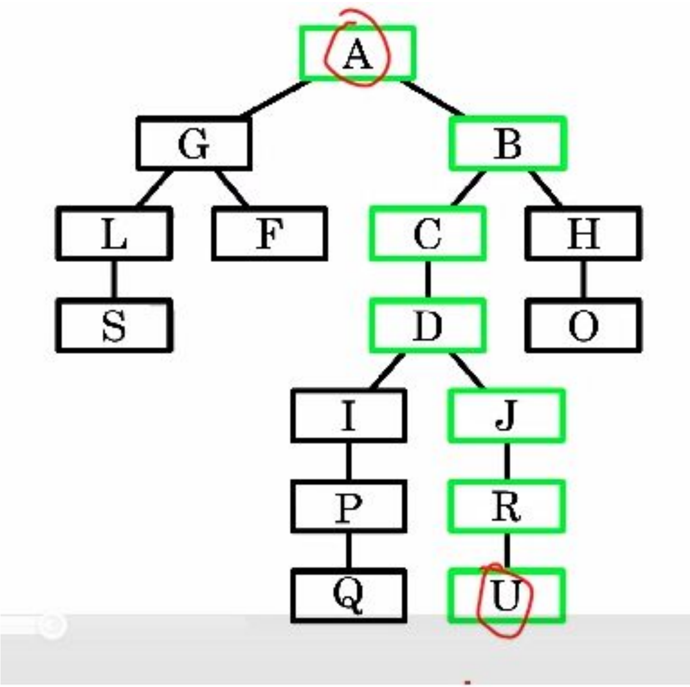



```
route(a,g).
route(g,l).
route(l,s).
...
travel(A,A).
travel(A,C) :- route(A,B),travel(B,C).

solve :- start(A),finish(B), travel(A,B).
```

```
start(a).
finish(u).
```

Sample implementation (simple, given graph)

Graph Search



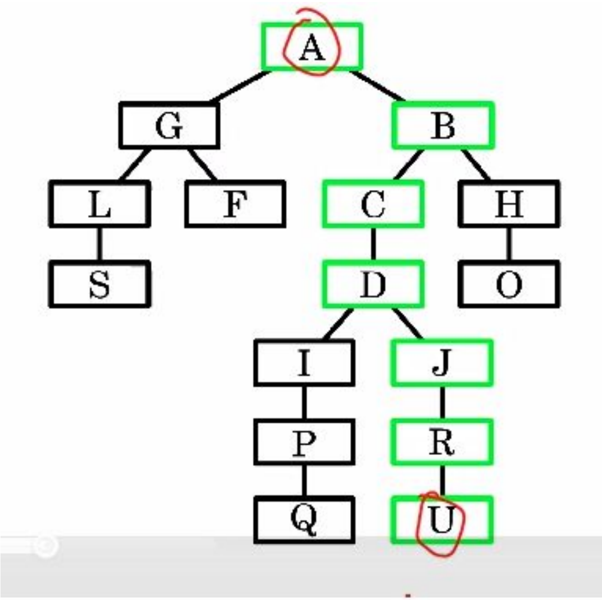

```
route(a,g).
route(g,l).
route(l,s).
...
travel(A,A).
travel(A,C) :- route(A,B),travel(B,C).

solve :- start(A),finish(B), travel(A,B).
```

```
start(a).
finish(u).
```

```
:- op(700, xfx, arc).
a arc g.
a arc b.
b arc c.
b arc h.
c arc d.
d arc i.
d arc j.
g arc f.
g arc l.
h arc o.
i arc p.
j arc r.
l arc s.
p arc q.
r arc u.

:- op(700, xfx, path).
X path Y :- X arc Y.
X path Y :- X arc W, W path Y.
```

```
arcs(a,[b,g]).
arcs(b,[c,h]).
arcs(c,[d]).
arcs(d,[i,j]).
arcs(g,[l,f]).
arcs(h,[o]).
arcs(i,[p]).
arcs(j,[r]).
arcs(l,[s]).
arcs(p,[q]).
arcs(r,[u]).

X arc Y :- arcs(X,Nodes),
           member(Y,Nodes).
```

Graph Search



Accumulating the path (or cost...):

```
:- op(700, xfx, arc).
a arc g.
a arc b.
b arc c.
b arc h.
c arc d.
d arc i.
d arc j.
g arc f.
g arc l.
h arc o.
i arc p.
j arc r.
l arc s.
p arc q.
r arc u.

% path(+Start,+Finish,-Path) succeeds if...
path(Start,Finish,Path) :- path_acc(Start,Finish,[],Path).

% path_acc(+Start,+Finish,+PathSoFar,-FullPath)
path_acc(X,Finish,Acc,Path) :- X arc Finish,
                               reverse([Finish,X|Acc],Path).

path_acc(X,Finish,Acc,Path) :- X arc Z,
                               path_acc(Z,Finish,[X|Acc],Path).
```

(1) Base case: the first `path_acc` clause.

(2) Recursive case: the second `path_acc` clause.

Accumulating here

Copying to solution here (via reverse/2)

Next time

Videos

- Difference
- Empty difference lists
- Difference list example

Q: you generally put the base case rule first e.g. Split([], [], []) - wouldn't it be more efficient to put this last since it is less likely? (fewer unifications)

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A: you would make a small saving if you only wanted one answer but more answers were possible. But you would still have all the choice points. Remember that order often matters when you have cut.

Q: Do we need to be able to compare Prolog to ML and functional programming? As a third year 50%er that was all a while ago...

Q: Do we need to be able to compare Prolog to ML and functional programming? As a third year 50%er that was all a while ago...

A: I won’t ask you to write ML in the exam. (But I would expect you to recall the concepts of the ML course as a general principle - what’s the point of your degree otherwise?)

Q: What is the underlying difference between a rule and a compound term? Same syntax right?

Q: What is the underlying difference between a rule and a compound term? Same syntax right?

A: a compound term is a ‘term’ in first order logic, a rule is ‘formula’ in first order logic.

Q: Is single cut rule bad practice?

```
last(H,[H]).
last(X,[_|T]) :- last(X,T).
```

This pointlessly backtracks after finding the answer.

So change axiom to: `last(H,[H]) :- !.`

Q: Is single cut rule bad practice?

```
last(H,[H]).
last(X,[_|T]) :- last(X,T).
```

This pointlessly backtracks after finding the answer.

So change axiom to: `last(H,[H]) :- !.`

A: It's fine to put a cut on a fact. The! Thing! To! Avoid! Is! Putting! One! Everywhere!

Next time

Videos

- Difference
- Empty difference lists
- Difference list example

Challenge: Write a tic-tac-toe (noughts and crosses) AI

What's the first step?

Challenge: Write a tic-tac-toe (noughts and crosses) AI

What predicate will you write and when will it succeed

```
% nextMove(Before,Player,After) succeeds if After represents the
% next state of the board after Player has made a move from state
% Before
```

Next step?

Challenge: Write a tic-tac-toe (noughts and crosses) AI

Choose a representation for the board...

Challenge: Write a tic-tac-toe (noughts and crosses) AI

Suggestion: represent each board position as a number 1 to 9, represent the state of the board as the list of moves that have been made, e.g. [move(5,x),move(1,o)].

Challenge: Write a tic-tac-toe (noughts and crosses) AI

Now try to implement `nextMove(Before,Player,After)`

Represent moves as move(Position,Player)

Represent the game state as a list of moves that have been made

Challenge: Write a tic-tac-toe (noughts and crosses) AI

```
pos(Index) :- member(Index,[1,2,3,4,5,6,7,8,9]).

used(I,[move(I,_)|_]).
used(I,[_|T]) :- used(I,T).

nextMove(Before,P,[move(Index,P)|Before]) :-
       pos(Index), \+used(Index,Before).
```

How could we make it smarter?

Teach it heuristics about good moves

- Prefer a corner at the start
- Take the middle if the corner is gone
- Win if you can
- Block the other player from winning if you can